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Atomic Coupler with Two-Mode Squeezed Vacuum States

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ABSTRACT

We investigate the entanglement transfer from the two-mode squeezed state (TMS) to the atomic system by studying the dependence of the negativity on the coupling between the modes of the waveguides. This study is very important since the entanglement is an important feature which has no classical counterpart and it is the main resource of quantum information processing. We use a linear coupler which is composed of two waveguides placed close enough to allow exchanging energy between them via evanescent waves. Each waveguide includes a localized atom.

Keywords: atomic quantum coupler; entanglement;two-mode squeezed state

1. Introduction

Linear coupler is a device composed of two or more waveguides placed close enough to allow exchanging energy between them via evanescent waves(Jensen, 1982). Each waveguide includes a localized atom. The flow of the energy can be controlled by the intensity of the input fields as well as the atomic configuration. The Hamiltonian of the atomic coupler consists of the sum of the free Hamiltonian and interaction Hamiltonian. The coupling between the modes of the waveguides occurs through the evanescent wave. This coupling shows many interesting behaviors for the linear atomic coupler. We assume in our model that the couplings are modeled by the Jaynes-Cummings interactions(Jaynes , 1963).

Our focus will be on the case when the initial state of the field is given by two-mode squeezed vacuum state(Caves, 1985)

. It is well known that the output of a non-degenerate optical parametric oscillator generates a two-mode squeezed state. It is also worth mentioning that if we are interested in one mode of the two-mode squeezed state, the probability of finding n photon in this mode is equivalent to finding n photons is thermal field.

Faisal and Wahiddin has developed an atomic quantum coupler (AQC) which consists of two different modes propagating into two different waveguides (Faisal , 2010). These two waveguides are placed close enough and each of them includes one trapped atom. The interaction between the mode and the atom is modeled by Jaynes-Cummings model (JCM).

We will extend their work and investigate the entanglement transfer from the two-mode squeezed state to the atomic system by studying the dependence of the negativity (Peres, 1996) on the coupling between the modes of the waveguides. This study is very important since the entanglement plays an important role in quantum information processing. We expect that our model is reliable for designing atomic coupler as well as understanding the entanglement transfer between the modes and the atoms.

2. Model

The model consists of two waveguides, each of which includes a localized and/or a trapped atom. The waveguides are placed close enough to each other to allow interchanging energy between them. The two atoms (in the different waveguides) are located very adjacent to each other. In each waveguide one mode propagates along and interacts with the atom inside in a standard JCM way. The atom-mode in each waveguide interacts with the other one via the evanescent wave. In the rotating wave approximation (RWA) the Hamiltonian can be expressed as:

$$\frac{\hat{H}}{\hbar} = \hat{H}_0 + \hat{H}_I, \quad \hat{H}_0 = \sum_{j=0}^{2} \omega_j \hat{a}_j^\dagger \hat{a}_j + \frac{\omega_a}{2}(\hat{\sigma}_z^{(1)} + \hat{\sigma}_z^{(2)}), \tag{1}$$

$$\hat{H}_I = \sum_{j=1}^{2} \lambda_j (\hat{a}_j \hat{\sigma}_+^{(j)} + \hat{a}_j^\dagger \hat{\sigma}_-^{(j)}) + \lambda_3 (\hat{a}_1 \hat{a}_2^\dagger \hat{\sigma}_+^{(1)} \hat{\sigma}_-^{(2)} + \hat{a}_1^\dagger \hat{a}_2 \hat{\sigma}_-^{(1)} \hat{\sigma}_+^{(2)}), \tag{2}$$

where $\hat{H}_0$ and $\hat{H}_I$ are the free and the interaction parts of the Hamiltonian, $\hat{\sigma}_\pm^{(j)}$ and $\hat{\sigma}_z^{(j)}$ are the Pauli spin operators of the jth atom ($j = 1, 2$); $\hat{a}_j$ ($\hat{a}_j^\dagger$) is the annihilation (creation) operator of the jth-mode with the frequency ω_j and ω_a is the atomic transition frequency (we consider that the frequencies of the two atoms are equal) and λ_1 (λ_2) is the atom-field coupling constant in the first (second) waveguide in the framework of the JCM. The interaction between the modes in the two waveguides occurs through the evanescent wave with the coupling constant λ_3. This term is the only one, which is conservative and can execute switching between the two waveguides. Thus it plays an essential role in the behavior of the AQC.

The interaction of two two-level atoms with the two modes has been considered in the optical cavity earlier(Faisal, 2008) , (Eberly, 2007). The quantum properties of the system of two two-level atoms interacting with the two nondegenerate cavity modes when the atoms and the field are initially in the atomic superposition states and the pair-coherent state has been investigated in (Faisal, 2008).

The two-mode squeezed vacuum state (TMS) has the form:

$$|r\rangle = \frac{1}{\cosh(\frac{r}{2})} \sum_{n=0}^{\infty} (\tanh r/2)^n \exp(i\phi_2 n)|n, n\rangle, \tag{3}$$

where ϕ_2

is the phase. Throughout the calculation we use the generic state

$$|\psi_f(0)\rangle = \sum_{n=0}^{\infty} C_n |n, n\rangle, \tag{4}$$

and for the sake of simplicity we take $\phi_j = 0, j = 1, 2$.

We use the technique given in (**?**) for solving the dynamical equation of the system. This technique is sensitive for the initial atomic states. Therefore we have to solve the schrödinger's equation for times. We write down the analytic solution for these cases as follows.

For the atomic states $|e_1, e_2\rangle$ the wavefunction takes the form

$$\begin{aligned} \mid \Psi_1(t)\rangle &= \sum_{n=0}^{\infty} C_n \Big[X_1^{(1)}(t,n,n) \mid e_1, e_2, n, n\rangle + X_2^{(1)}(t,n,n) \mid e_1, g_2, n, n+1\rangle \\ &+ X_3^{(1)}(t,n,n) \mid g_1, e_2, n+1, n\rangle + X_4^{(1)}(t,n,n) \mid g_1, g_{2,} n+1, n+1\rangle\Big] \quad (5) \end{aligned}$$

From the initial conditions, the exact forms of the coefficients X_j can be expressed as:

$$\begin{aligned} X_1^{(1)}(t,n,n) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(1)})\left[\cos(t\Omega_-^{(1)}) - i\frac{c_2^{(1)}}{2\Omega_-^{(1)}}\sin(t\Omega_-^{(1)})\right] \\ &+\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(1)})\left[\cos(t\Omega_+^{(1)}) + i\frac{c_2^{(1)}}{2\Omega_+^{(1)}}\sin(t\Omega_+^{(1)})\right], \quad (6) \\ X_2^{(1)}(t,n,n) &= -\frac{i\sin(t\Omega_-^{(1)})}{2\Omega_-^{(1)}}[\lambda_2-\lambda_1]\sqrt{n+1}\exp(i\frac{t}{2}c_2^{(1)}) \\ &-\frac{i\sin(t\Omega_+^{(1)})}{2\Omega_+^{(1)}}[\lambda_2+\lambda_1]\sqrt{n+1}\exp(-i\frac{t}{2}c_2^{(1)}), \quad (7) \\ X_3^{(1)}(t,n,n) &= \frac{i\sin(t\Omega_-^{(1)})}{2\Omega_-^{(1)}}[\lambda_2-\lambda_1]\sqrt{n+1}\exp(i\frac{t}{2}c_2^{(1)}) \\ &-\frac{i\sin(t\Omega_+^{(1)})}{2\Omega_+^{(1)}}[\lambda_2+\lambda_1]\sqrt{n+1}\exp(-i\frac{t}{2}c_2^{(1)}), \quad (8) \\ X_4^{(1)}(t,n,n) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(1)})\left[-\cos(t\Omega_-^{(1)}) + i\frac{c_2^{(1)}}{2\Omega_-^{(1)}}\sin(t\Omega_-^{(1)})\right] \\ &+\frac{1}{2}\exp(-i\frac{t}{2}c_2^{(1)})\left[\cos(t\Omega_+^{(1)}) + i\frac{c_2^{(1)}}{2\Omega_+^{(1)}}\sin(t\Omega_+^{(1)})\right], \quad (9) \end{aligned}$$

where

$$c_1^{(1)} = 2\lambda_1\lambda_2(n+1),\ c_2^{(1)} = \lambda_3(n+1),\ \Omega_{\pm}^{(1)} = \sqrt{\frac{c_2^{(1)2}}{4} + (n+1)(\lambda_1 \pm \lambda_2)^2}. \quad (10)$$

For the atomic states $|g_1, g_2\rangle$ the wavefunction takes the form

$$\begin{aligned} \mid \Psi_2(t)\rangle &= \sum_{n=0}^{\infty} C_n \Big[X_1^{(2)}(t,n,n) \mid e_1, e_2, n-1, n-1\rangle + X_2^{(2)}(t,n,n) \mid e_1, g_2, n-1, n\rangle \\ &+ X_3^{(2)}(t,n,n) \mid g_1, e_2, n, n-1\rangle + X_4^{(2)}(t,n,n) \mid g_1, g_2, n, n\rangle \Big], \end{aligned} \quad (11)$$

From the initial conditions, the exact forms of the coefficients X_j can be expressed as:

$$\begin{aligned} X_1^{(2)}(t,n,n) &= \frac{1}{2}\exp(i\frac{t}{2}c_2^{(2)}) \left[-\cos(t\Omega_-^{(2)}) + i\frac{c_2^{(2)}}{2\Omega_-^{(2)}} \sin\left(t\Omega_-^{(2)}\right) \right] \\ &+ \frac{1}{2}\exp\left(-i\frac{t}{2}c_2^{(2)}\right) \left[\cos\left(t\Omega_+^{(2)}\right) + i\frac{c_2^{(2)}}{2\Omega_+^{(2)}} \sin\left(t\Omega_+^{(2)}\right) \right], \end{aligned} \quad (12)$$

$$\begin{aligned} X_2^{(2)}(t,n,n) &= \frac{i\sin\left(t\Omega_-^{(2)}\right)}{2\Omega_-^{(2)}} [\lambda_2 - \lambda_1]\sqrt{n}\exp\left(i\frac{t}{2}c_2^{(2)}\right) \\ &- \frac{i\sin\left(t\Omega_+^{(2)}\right)}{2\Omega_+^{(2)}} [\lambda_2 + \lambda_1]\sqrt{n}\exp\left(-i\frac{t}{2}c_2^{(2)}\right), \end{aligned} \quad (13)$$

$$\begin{aligned} X_3^{(2)}(t,n,n) &= \frac{-i\sin\left(t\Omega_-^{(2)}\right)}{2\Omega_-^{(2)}} [\lambda_2 - \lambda_1]\sqrt{n}\exp\left(i\frac{t}{2}c_2^{(2)}\right) \\ &- \frac{i\sin\left(t\Omega_+^{(2)}\right)}{2\Omega_+^{(2)}} [\lambda_2 + \lambda_1]\sqrt{n}\exp\left(-i\frac{t}{2}c_2^{(2)}\right), \end{aligned} \quad (14)$$

$$\begin{aligned} X_4^{(2)}(t,n,n) &= \frac{1}{2}\exp\left(i\frac{t}{2}c_2^{(2)}\right) \left[\cos\left(t\Omega_-^{(2)}\right) - i\frac{c_2^{(2)}}{2\Omega_-^{(2)}} \sin\left(t\Omega_-^{(2)}\right) \right] \\ &+ \frac{1}{2}\exp\left(-i\frac{t}{2}c_2^{(2)}\right) \left[\cos\left(t\Omega_+^{(2)}\right) + i\frac{c_2^{(2)}}{2\Omega_+^{(2)}} \sin\left(t\Omega_+^{(2)}\right) \right], \end{aligned} \quad (15)$$

where

$$c_1^{(2)} = 2\lambda_1\lambda_2 n,\ c_2^{(2)} = \lambda_3 n,\ \Omega_\pm^{(2)} = \sqrt{\frac{c_2^{(2)2}}{4} + n(\lambda_1 \pm \lambda_2)^2}. \quad (16)$$

3. Entanglement Transfer

Entanglement plays an important role in quantum teleportation (Bennet, 1993), quantum cryptography(Ekert, 1991) and quantum computation(Braunstein, 1999),(Eisert, 2000). There are different criteria for measuring entanglement for two qubits, e.g. concurrence(Wootters, 1998) and negativity(Peres, 1996).

In our paper we use the negativity to measure the degree of entanglement. It is given by

$$E = \max\left(0, -2\sum_i \mu_{i-}\right), \quad (17)$$

where the sum is taken over the negative eigenvalues μ_{i-} of the partial transposition of the density matrix $\hat{\rho}$ of the system The value $E = 1$ corresponds to maximum entanglement between the atoms whilst $E = 0$ describes completely separated atoms. In our model the only eigenvalue that can be negative is

$$\mu_- = \frac{1}{2}\left(\rho_{22} + \rho_{33} - \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{14}|^2}\right),$$

where the states $|1\rangle = |e_1, e_2\rangle$, $|2\rangle = |e_1, g_2\rangle$, $|3\rangle = |g_1, e_2\rangle$, and $|4\rangle = |g_1, g_2\rangle$. So, the entanglement transfer occurs only when $\mu_- < 0$.

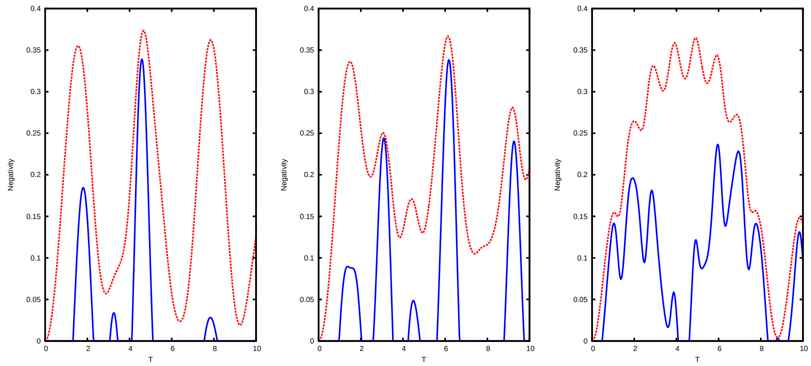

Figure 1: The negativity as a function of time for the parameters $r = 1$ and $(\lambda_1, \lambda_2, \lambda_3) = (1, 1, 0)$ for left, (1,1,1) for centre, and (1,1,5) for right. The solid(dashed) line is for the atoms are initially in the excited(ground) states.

The Figure 1 shows the negativity as a function of time when the atoms are initially in the exited state (solid line) or in the ground states (dashed line). The atoms are entangled most of the time if the atoms started initially in the ground states. However, the atoms are separated most of the time when they started at the excited states. It is evident that when the coupling parameter λ_3 increases, the entanglement of the atoms occurs most of the time irrespective whether the atoms are initially in the ground or excited states.

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